Composite Gluons and Effective Nonabelian Gluon Dynamics in a Unified Spinor-Isospinor Preon Field Model

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The model is defined by a selfregularizing nonlinear preon field equation and all observable (elementary and non-elementary) particles are assumed to be bound (quantum) states of the fermionic preon fields. In particular electroweak gauge bosons are two-particle composites, leptons and quarks are three-particle composites, and gluons are six-particle composites. Electroweak gauge bosons, leptons and quarks and their effective interactions etc. were studied in preceding papers. In this paper gluons and their effective dynamics are discussed. Due to the complications of a six-particle bound state dynamics the formation of gluons is performed in two steps: First the effective dynamics of three-particle composites (quarks) is derived, and secondly gluons are fusioned from two quarks respectively. The resulting effective gluon dynamics is a non-abelian SU(3) dynamics, i.e. this local gauge dynamics is produced by the properties of the composites and need not be introduced in the original preon field equation. Mathematically these results are achieved by the application of functional quantum theory to the model under consideration and subsequent evaluation of weak mapping procedures, both introduced in preceding papers.

PACS 11.10 Field theory.
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PACS 12.35 Composite models of particles.

Introduction

The complexity of experimental results on the lepton-quark level has led to speculations about subquark and sublepton models, cf. the review article of Lyons [1]. The use of such models is justified if they are less complicated than models on the lepton-quark level and if they allow a well-defined quantum field theoretical formulation. An extremely simple model which satisfies both these conditions has been elaborated by the author [2, 3, 4, 5] and coworkers, Grosser and Lauxmann [6], Grosser, Hailer, Hornung, Lauxmann and Stumpf [7], and Grosser [8]. The model is defined by a selfregularizing nonlinear spinor-isospinor preon field equation and all observable (elementary and non-elementary) particles are assumed to be bound states, i.e. composites of the quantized preon field. The basic physical ingredients of this model are realizations, combinations and further developments of ideas of de Broglie [9], Bopp [10] and Heisenberg [11], whereas the mathematical tools for the quantitative evaluation of the model are novel developments. A review of the development of the compositeness hypothesis is contained in [5]. An extensive discussion of the results and of the mathematical techniques with respect to the model under consideration is contained in [3]. The general mathematical tool is the so-called functional quantum theory, a formulation of quantum field theory which prefers to work mainly with states and not with operators as was elaborated and advocated by the author [12].

After having formulated the model and obtained a first insight into the formation of bound states etc. in [2, 6, 7, 8] the most urgent problem is the investigation of effective interactions between relativistic composite particles and the analysis of their structure as preon bound states. In particular it has to be proved that relativistic composite particles representing observable "elementary" particles satisfy in certain approximations the corresponding gauge theories, etc., which govern the reactions of these particles if they are considered to be elementary and point-like. Within the framework of functional quantum theory the mathematical method to handle such problems is the "weak mapping" procedure which was introduced in [3] and extensively discussed and compared with the current "strong

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mapping" procedure in [3, 4] and [5]. By means of this method it was demonstrated in [3] that in the low energy range for composite scalar bosons and composite spin 1/2 fermions a Yukawa theory results as effective field theory, while in [4] for composite vector bosons a non-abelian SU(2) Yang-Mills gauge theory was derived (without assuming a local gauge invariance in the underlying preon field model!). Finally in [5] the preon structure of leptons and Han-Nambu quarks (with color) was derived and it was demonstrated that the effective coupling of these particles to electroweak (composite) gauge bosons of [4] yields the Weinberg-Salam coupling in the low energy range. Furthermore, in the high energy range such effective theories are modified by the appearance of form-factors, cf. [3].

By means of the same techniques we investigate in this paper the structure and the effective dynamics of gauge gluons, i.e. we analyse an SU(3) non-abelian composite vector boson effective field theory. In contrast to electroweak gauge bosons (two-preon composites) and leptons and Han-Nambu quarks (three-preon composites) it is necessary to assume the gauge gluons to be six-preon composites. For the treatment of this problem it is advantageous to perform a two-step formation. In the first step three-preon clusters are formed while in the second step two such clusters are fusioned to give a six-preon composite. This procedure is analogous to the treatment of such problems in nuclear cluster physics which was mainly developed by Wildermuth et al. [13], Schmid [14], and Kramer [15], and in nuclear quark physics, cf. Faessler [16] and Müther et al. [17]. In contrast, however, to this nonrelativistic cluster approach, in relativistic quantum field theory weak mapping and cluster physics need a more sophisticated elaboration by means of functional quantum theory in order to achieve any result.

The necessity of considering gauge gluons as six-preon composites coincides with proposals of the gauge gluon structure in other simple models, Harari [18], Shupe [19], Elbaz [20]. A discussion of these models which need for their field theoretic formulation the explicit introduction of elementary hypercolor and color gauge bosons was performed in [5] and need not be repeated here. It should, however, be emphasized that although for fermions and gluons the sub-particle numbers are equal, the physical content of both theories is quite different: while the latter authors arrived at a complicated gauge theory on the preon level, our model is a simple self-coupled preon field. In addition, of course, our preons have integer charges and the gluons correspond to Han-Nambu gluons. It might be doubted whether Han-Nambu quarks (and gluons) are ideal parametrizations of quark theories. But our first aim is to reveal gauge theories as composite particle reactions; later on details can be modified and improved.

1. Fundamentals of the Model

The unified preon field model which is assumed to be the basis of the theory is defined by the second-order derivative nonlinear field equation

$$[(-i\gamma^\mu\partial_\mu + m_1)(-i\gamma^\varrho\partial_\varrho + m_2)]_{\alpha\beta}\,\psi_\beta(x) = g\,V_{\alpha\beta\gamma\delta}\,\psi_\beta(x)\,\bar\psi_\gamma(x)\,\psi_\delta(x)\,, \tag{1.1}$$

where the indices $\alpha, \beta, \ldots$ are superindices describing spin and isospin. Due to the mass term in (1.1) the corresponding spinor field has to be a Dirac-spinor-isospinor.

In contrast to the non-renormalizability of first order derivative nonlinear spinorfield equations and the difficulties connected with this property, the model (1.1) exhibits self-regularization, relativistic invariance and locality for common canonical quantization. Due to the self-regularization the model is renormalizable, but we need not make use of this property on account of our non-perturbative calculation techniques.

For the further evaluation equation (1.1) has to be decomposed into an equivalent set of first order derivative equations. It was proved by the author [2] and Grosser [8] that the set of nonlinear equations $r = 1, 2$

$$(-i\gamma^\mu\partial_\mu + m_r)_{\alpha\beta}\,\varphi_{\beta r}(x) = g\,\lambda_r \sum_{stu} V_{\alpha\beta\gamma\delta}\,\varphi_{\beta s}(x)\,\bar\varphi_{\gamma t}(x)\,\varphi_{\delta u}(x) \tag{1.2}$$

is connected with (1.1) by a biunique map where this map is defined by the compatible relations

$$\begin{aligned}\psi_\alpha(x) &= \varphi_{\alpha 1}(x) + \varphi_{\alpha 2}(x)\,,\\ \varphi_{\alpha 1}(x) &= \lambda_1(-i\gamma^\mu\partial_\mu + m_2)_{\alpha\beta}\,\psi_\beta(x)\,,\\ \varphi_{\alpha 2}(x) &= \lambda_2(-i\gamma^\mu\partial_\mu + m_1)_{\alpha\beta}\,\psi_\beta(x)\end{aligned} \tag{1.3}$$

with $\lambda_r := (-1)^r\,(2\Delta m)^{-1}$ and $\Delta m = 1/2\,(m_1 - m_2)$.

According to [4, 5] the vertex operator must have the form

$$V_{\alpha\beta\gamma\delta} = \tfrac{1}{2} \sum_{h=1}^{2} (v^h_{\alpha\beta} v^h_{\gamma\delta} - v^h_{\alpha\delta} v^h_{\gamma\beta}) \tag{1.4}$$

with $\alpha \equiv (\alpha, A) \equiv$ (spinor, isospinor index) etc. and

$$v^h_{\alpha\beta} \equiv \hat{v}^h_{\alpha\beta}\, \delta_{AB}\,; \quad h = 1, 2\,;$$
$$\hat{v}^1_{\alpha\beta} := \delta_{\alpha\beta}\,; \quad \hat{v}^2_{\alpha\beta} := i\gamma^5_{\alpha\beta}\,. \tag{1.5}$$

This kind of chiral invariant coupling was used in the symmetry breaking first order derivative nonlinear spinorfield model of Nambu and Jona-Lasinio [21] and is equivalent to the vector and pseudovector coupling of Heisenberg's first order derivative nonlinear spinorfield equation [1] due to the Fierz theorem. Although the kinetic part of (1.2) is not chiral invariant, this kind of coupling is needed to obtain non-abelian effective field theories for composite particles.

For the further elaboration it is convenient to replace the adjoint spinors by the charge conjugated spinor [4, 5]. The charge conjugated spinor (isospinor) is defined by

$$\varphi^c_{Aj\nu} := C^{-1}_{\nu\nu'}\, \bar{\varphi}_{Aj\nu'} \tag{1.6}$$

and introducing the superspinors

$$\varphi_{Aj\alpha 1} := \varphi_{Aj\alpha}\,,$$
$$\varphi_{Aj\alpha 2} := \varphi^c_{Aj\alpha}\,, \tag{1.7}$$

we can combine (1.2) and its charge conjugated equation into one equation [4, 5]

$$\sum_{Z_2} (D^\mu_{Z_1Z_2}\partial_\mu - m_{Z_1Z_2})\, \varphi_{Z_2}(x) = \sum_{h Z_2Z_3Z_4} U^h_{Z_1Z_2Z_3Z_4}\, \varphi_{Z_2}(x)\, \varphi_{Z_3}(x)\, \varphi_{Z_4}(x) \tag{1.8}$$

with $Z := (\alpha, A, i, \Lambda)$ and

α = spinor index ($\alpha = 1, 2, 3, 4$),
A = isospinor index ($A = 1, 2$),
i = auxiliary field index ($i = 1, 2$),
Λ = superspinor index ($\Lambda = 1, 2$), (1.9)

where the following definitions are used

$$D^\mu_{Z_1Z_2} := i\gamma^\mu_{\alpha_1\alpha_2}\, \delta_{A_1A_2}\, \delta_{i_1i_2}\, \delta_{\Lambda_1\Lambda_2}\,,$$
$$m_{Z_1Z_2} := m_{i_1}\, \delta_{\alpha_1\alpha_2}\, \delta_{A_1A_2}\, \delta_{i_1i_2}\, \delta_{\Lambda_1\Lambda_2}\,, \tag{1.10}$$
$$U^h_{Z_1Z_2Z_3Z_4} := g\,\lambda_{i_1}\, \hat{v}^h_{\alpha_1\alpha_2} \delta_{A_1A_2} \delta_{\Lambda_1\Lambda_2} (\hat{v}^h C)_{\alpha_3\alpha_4} \delta_{A_3A_4} \delta_{\Lambda_3 1}\, \delta_{\Lambda_4 2}\,.$$

The quantum states of the model (1.1) or (1.8), respectively, are described by state functionals $|\mathfrak{T}[j, a]\rangle$ with respect to the states $\{|a\rangle\}$ where $j \equiv j_Z(x)$ are sources with corresponding Z-indices. For concrete calculations it is necessary to introduce normal transforms by $|\mathfrak{T}\rangle = \mathscr{F}_0[j]\,|\mathfrak{F}\rangle$ and the energy representation of the spinorfield in terms of state functionals. Both procedures were discussed in detail in [7] and yield a functional equation for $|\mathfrak{F}\rangle$. In this equation the limit to a one-time description can be performed, so that eventually the following equation results

$$p_0 |\mathfrak{F}\rangle = \sum_{ZZ_1Z_2} \int j_Z(\mathbf{r})\, i D^0_{ZZ_1} [D^k_{Z_1Z_2} \partial_k - m_{Z_1Z_2}]\, \partial_{Z_2}(\mathbf{r})\, \mathrm{d}^3r\, |\mathfrak{F}\rangle + \sum_{hZZ_1Z_2Z_3Z_4} \int j_Z(\mathbf{r})\, i D^0_{ZZ_1} U^h_{Z_1Z_2Z_3Z_4}\, d_{Z_4}(\mathbf{r})\, d_{Z_3}(\mathbf{r})\, d_{Z_2}(\mathbf{r})\, \mathrm{d}^3r\, |\mathfrak{F}\rangle\,, \tag{1.11}$$

where

$$d_Z(\mathbf{r}) := \partial_Z(\mathbf{r}) - \sum_{Z'} \int F_{ZZ'}(\mathbf{r}, \mathbf{r}')\, j_{Z'}(\mathbf{r}')\, \mathrm{d}^3r'\,. \tag{1.12}$$

Equation (1.11) can be written in shorthand if the abbreviations

$$K_{I_1I_2} := K_{Z_1Z_2}(\mathbf{r}_1, \mathbf{r}_2) := i \sum_Z D^0_{Z_1Z} (D^k_{ZZ_2} \partial^1_k - m_{ZZ_2})\, \delta(\mathbf{r}_1 - \mathbf{r}_2) \tag{1.13}$$

$$W^h_{I_1I_2I_3I_4} := W^h_{Z_1Z_2Z_3Z_4}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) := i \sum_Z D^0_{Z_1Z} U^h_{ZZ_2Z_3Z_4}\, \delta(\mathbf{r}_1 - \mathbf{r}_2)\, \delta(\mathbf{r}_1 - \mathbf{r}_3)\, \delta(\mathbf{r}_1 - \mathbf{r}_4) \tag{1.14}$$

are introduced. Then (1.11) reads

$$p_0 |\mathfrak{F}\rangle = \sum_{I_1I_2} j_{I_1} K_{I_1I_2}\, \partial_{I_2} |\mathfrak{F}\rangle + \sum_{hI_1I_2I_3I_4} j_{I_1} W^h_{I_1I_2I_3I_4}\, d_{I_4} d_{I_3} d_{I_2} |\mathfrak{F}\rangle =: \mathscr{H} |\mathfrak{F}\rangle\,. \tag{1.15}$$

We assume the spinorfield interaction term to be normalordered. Then the local terms $F_{I_2 I_3}\partial_{I_4}$ etc. drop out and we obtain a singularity free interaction from (1.15). The special form of this evaluated interaction term of (1.15) was given in [4]. It reads

$$\begin{aligned} d_{I_4} d_{I_3} d_{I_2} = \partial_{I_4}\partial_{I_3}\partial_{I_2} &- \sum_K [F_{I_4K}\, j_K\, \partial_{I_3}\partial_{I_2} - F_{I_3K}\, j_K\, \partial_{I_2}\partial_{I_4} - F_{I_2K}\, j_K\, \partial_{I_4}\partial_{I_3}] \\ &+ \sum_{KK'} [F_{I_4K} F_{I_3K'}\, j_K j_{K'}\, \partial_{I_2} + F_{I_3K} F_{I_2K'}\, j_K j_{K'}\, \partial_{I_4} + F_{I_2K} F_{I_4K'}\, j_K j_{K'}\, \partial_{I_3}] \\ &- \sum_{KK'K''} F_{I_4K} F_{I_3K'} F_{I_2K''}\, j_K j_{K'} j_{K''}\,, \end{aligned} \tag{1.16}$$

where $F_{II'}$ is the antisymmetrized one-time limit of the preonfield causal propagator which is used in (1.12).

2. Dressed Composite Fermion States

The formation of gluons as six-preon composites is a rather complicated problem if the six-preon states are treated directly. In order to avoid such difficulties we resolve the formation of these vector bosons into a two-step fusion mechanism: We first consider the fusion of three-preon fermionic composites and afterwards by means of the effective interactions of these composite fermions we obtain by fusion of two three-fermion composites the gluons. But irrespective of whether a two-step fusion mechanism is applied or not, the calculation of the effective interactions between preon composites for $n \geqq 3$ requires the inclusion of the preonic polarization cloud from the beginning. While for the case $n = 2$ the effective interactions can be calculated without any regard to the preonic polarization cloud, for $n \geqq 3$ composites their preonic polarization cloud is responsible for their mutual effective interactions and constitutes them. In this way, the $n = 2$ composites play a distinguished role compared with $n \geqq 3$ composites, as for $n = 2$ composites we have direct "hard core" interactions and the polarization cloud actions and interactions can be recast into the corresponding effects on the level of the effective interactions, while the $n \geqq 3$ composites lead to "peripheral" interactions, i.e. interactions which are due to the overlap of polarization clouds. Hence the physics of the $n = 2$ composites should be more elementary than the physics for $n \geqq 3$ composites, a fact which will become obvious perhaps on a more elaborated stage of the theory. For the present problem we simply take into account the necessity of the use of dressed particle states for the weak mapping procedure and consider the case $n = 3$. For this case the functional dressed particle operators of three-preon composites are defined by the functional power series expansion

$$f_{3,K} = \sum_{N=0}^{\infty} C_{K,2N+3}^{I_1 \ldots I_{2N+3}}\, j_{I_1} \ldots j_{I_{2N+3}}\,, \tag{2.1}$$

where the set of coefficients $\{C_{K,2N+3}^{I_1 \ldots I_{2N+3}},\ N = 0 \ldots \infty\}$ are the "wave" functions of the corresponding states labeled by the quantum number index K, while the index 3 denotes the three-preon composites. A complete treatment would require the inclusion of higher number preon composites with functional operators $\{f_{3+2M,K},\ M = 1 \ldots \infty\}$ in order to fulfill all completeness relations etc. But for a first inspection we restrict ourselves to the set of operators $\{f_{3,K}\}$ of (2.1). For the further evaluation the dual states of the set $\{C_{K,2N+3}\}$ are required. Denoting these states by $\{R_{I_1 \ldots I_{2N+3}}^{K',2N+3},\ N = 0 \ldots \infty\}$ according to their definition we have

$$\sum_{N=0}^{\infty} R_{I_1 \ldots I_{2N+3}}^{K',2N+3}\, C_{K,2N+3}^{I_1 \ldots I_{2N+3}} = \delta_{K'K}\,. \tag{2.2}$$

If the corresponding physical states $|K\rangle$ or $|K'\rangle$, respectively, are positive definite, the dual states are left-hand solutions of the functional energy operator [2]. As for three-preon composites in the low energy limit and in the "hard core" approximation $N = 0$ the positive definiteness was demonstrated in [3] we assume that this condition is also satisfied for full dressing. Then the corresponding dual states can be directly calculated as left-hand solutions of the eigenvalue problem (1.15) and can thus be assumed to be explicitly known (at least in certain approximations).

For the application of such explicit left-hand solutions to the calculation of effective interactions of the corresponding particles a further property of them must be taken into account. If the quantum numbers $\{K, K'\}$ of the two sets $\{C_{K,2N+3}, N=0, 1, \dots\}$ and $\{R^{K,2N+3}, N=0,1,\dots\}$ are non-degenerate and if the dual set is given by the left-hand solutions of (1.15), then it can be shown that already the single terms of both these sets must satisfy the relations

$$R^{K',2N+3}_{I_1\dots I_{2N+3}} C^{I_1\dots I_{2N+3}}_{K,2N+3} = a_{2N+3}(K)\,\delta_{KK'}\,; \qquad N=0,1,\dots \tag{2.3}$$

with

$$\sum_{N=0}^{\infty} a_{2N+3}(K) = 1\,. \tag{2.4}$$

The proof of these relations rests on subsidiary conditions which result from the fact that the preon bound states have well defined quantum numbers, cf. [7]. For brevity we do not discuss this in detail.

Rather we investigate only the consequences. By means of these relations it is possible to represent the preon functional operator products $\{j_{I_1}\dots j_{I_M}, M=1\dots\}$ by the functional dressed particle operators $\{f_{3,K},\dots\}$, i.e. to form the inverse relation of (2.1). This inversion of (2.1) is required for the evaluation of the effective interactions.

The use of an infinite polarization cloud in (2.1) and in the corresponding dual functionals is an idealization. For the explicit calculations we take into account only those terms which are absolutely necessary for obtaining a non-trivial result. These are the terms $N=0,1,2$; i.e. we operate with the truncated sets $\{C_{K,2N+3}, N=0,1,2\}$ and $\{R^{K',2N+3}, N=0,1,2\}$. For these sets the inverse relations of (2.1) read

$$\begin{aligned} j_{I_1}j_{I_2}j_{I_3} &= \sum_{K'} R^{K',3}_{I_1I_2I_3} f_{3,K'}\,,\\ j_{I_1}j_{I_2}j_{I_3}j_{I_4}j_{I_5} &= \sum_{K'} R^{K',5}_{I_1I_2I_3I_4I_5} f_{3,K'}\,,\\ j_{I_1}j_{I_2}j_{I_3}j_{I_4}j_{I_5}j_{I_6}j_{I_7} &= \sum_{K'} R^{K',7}_{I_1I_2I_3I_4I_5I_6I_7} f_{3,K'}\,. \end{aligned} \tag{2.5}$$

These relations can be verified by substitution of (2.5) into (2.1) for the approximation $N=0,1,2$ and by use of (2.3) and (2.4). For the approximation $N=0$ the formulas (2.1), (2.2) and (2.5) are reduced to the case of undressed three-preon composites which was treated in preceding papers.

In the simplest non-trivial evaluation of composite particle interactions for composites $N \geqq 3$ only the dual set is required with dressing while for the original set only the bare wave functions are needed, i.e. we have to calculate only $\{C^{I_1I_2I_3}_{K,3}\}$ and $\{R^{K',3}, R^{K',5}, R^{K',7}\}$. The latter set can be approximately obtained from the eigenvalue equation (1.15) by a simple iteration procedure starting from the undressed "hard" core wave functions $C_{K,3}$ or $R^{K',3}$, resp.

The left-hand state functionals of (1.15) are defined to be the solutions of the following equation:

$$\langle \mathfrak{S}|\,p_0 = \langle \mathfrak{S}|\sum_{I_1I_2} K_{I_1I_2} j_{I_1}\partial_{I_2} + \langle \mathfrak{S}|\sum_{h\,I_1I_2I_3I_4} W^h_{I_1I_2I_3I_4} j_{I_1} d_{I_4} d_{I_3} d_{I_2}\,. \tag{2.6}$$

As usual we define the "hard core" energy operator (cf. [3]) by

$$\mathscr{H}_0 := \sum_{I_1I_2} K_{I_1I_2} j_{I_1}\partial_{I_2} - \sum_{K\,I_1[\alpha,\beta,\gamma]} W^h_{I_1I_\alpha I_\beta I_\gamma} j_{I_1} j_K F_{I_\alpha K}\partial_{I_\beta}\partial_{I_\gamma}\,, \tag{2.7}$$

where summation definition $[\alpha,\beta,\gamma] := \{(\alpha,\beta,\gamma) = (4,3,2), (3,2,4), (2,3,4)\}$ holds. We then rewrite (2.6) in the form

$$\langle \mathfrak{S}|\,(p_0 - \mathscr{H}_0) = \lambda \langle \mathfrak{S}|\sum_{h\,I_1I_2I_3I_4} W^h_{I_1I_2I_3I_4} j_{I_1}\partial_{I_4}\partial_{I_3}\partial_{I_2} + \dots \text{ further terms}\,, \tag{2.8}$$

where later on $\lambda=1$ is inserted, and assume $\langle \mathfrak{S}|$ to be given by a power series expansion

$$\langle \mathfrak{S}| = \sum_{N=0}^{\infty} \langle \mathfrak{S}_{2N+3}|\,\lambda^N\,, \tag{2.9}$$

where $\langle \mathfrak{S}_{2N+3}|$ is defined by

$$\langle \mathfrak{S}_{2N+3}| := \sum_{I_1\dots I_{2N+3}} R^{K',2N+3}_{I_1\dots I_{2N+3}} \partial_{I_1}\dots\partial_{I_{2N+3}}\,. \tag{2.10}$$

If in a first approximation only the explicit terms of (2.8) are taken into account, we obtain the recursion formulas for $N=0,1,2$

$$\begin{aligned} \langle \mathfrak{S}_3|\,(p_0-\mathscr{H}_0) &= 0\,,\\ \langle \mathfrak{S}_5|\,(p_0-\mathscr{H}_0) &= \langle \mathfrak{S}_3|\sum_{h\,I_1I_2I_3I_4} W^h_{I_1I_2I_3I_4} j_{I_1}\partial_{I_4}\partial_{I_3}\partial_{I_2}\,,\\ \langle \mathfrak{S}_7|\,(p_0-\mathscr{H}_0) &= \langle \mathfrak{S}_5|\sum_{h\,I_1I_2I_3I_4} W^h_{I_1I_2I_3I_4} j_{I_1}\partial_{I_4}\partial_{I_3}\partial_{I_2}\,, \end{aligned} \tag{2.11}$$

i.e. once the "hard core" wave functions $\langle\mathfrak{S}_3|$ are known, the higher polarization cloud wave functions can be obtained by simple integrations. We shall explicitly determine such quantities in Section 4.

The approximation (2.11) can be justified by some estimates: For the determination of the left-hand solutions we use the ansatz

$$R^{K,2N+3} = w_K^{2N+3} \chi^n_{K,2N+3}, \quad N = 0, 1, \dots, \tag{2.12}$$

where the $\chi^n_{K,2N+3}$ are normalized test-wavefunctions. Then the problem is reduced to the calculation of the w_K^{2N+3} which are the weighting coefficients of the contributions of the functions $R^{K,2N+3}$, $N = 0, 1, \dots$ to the whole polarization cloud. An estimate shows that the inclusion of the neglected terms in (2.11) does not qualitatively change the resulting values of the w_K^{2N+3}, $N = 0, 1, \dots$. For both cases we obtain $w_K^{2N+3} \approx m^{-N}$, i.e. the polarization cloud contributions rapidly converge ($m \equiv$ average preon mass).

3. Effective Composite Fermion Dynamics

The precondition for the formation of composite gluons by the fusion of two composite fermions is the derivation of the effective dynamics of these fermions. This can be achieved by means of the weak mapping procedure introduced in [3]. This procedure is defined by a transformation of the set of functional preon source operators $\{j_I\}$ into a set of (dressed) composite particle source operators $\{X_R\}$. In principle this transformation should comprise all kinds of dressed particle source operators which correspond to the complete set of dressed composite one-particle states of the theory (one-particle states with respect to the fact that only one composite particle of any kind is to be considered!). Only in this case one can expect to get a complete description of the complicated spectrum and interactions of the so-called "elementary" particles etc. On the other hand, it is convenient to treat various parts of the whole spectrum separately in order to get a first insight into the mechanism and the results of weak mapping procedures. Thus for the formation of composite gluons we leave aside all those composite particles which play no role for the fusion process and take into account only the required fermion states of Section 2.

The transformation of the preon source operators $\{j_I\}$ into fermion source operators $\{f_{3,K}\}$ induces a transformation of the set of functional states $\{|\mathfrak{F}\rangle\}$ as well as of the corresponding functional equation (1.15). These transformations are characterized by the invariance conditions

$$|\mathfrak{F}[j,a]\rangle = |\hat{\mathfrak{F}}[f,a]\rangle \tag{3.1}$$

and

$$\mathscr{H}\left[j, \frac{\delta}{\delta j}\right] = \hat{\mathscr{H}}\left[f, \frac{\delta}{\delta f}\right]. \tag{3.2}$$

For brevity we write in the following $f_{3,K} \equiv f_K$ etc. Then by the fermion transformation (2.1) the functional energy representation (1.15) goes over into the transformed equation

$$\begin{aligned} p_0|\hat{\mathfrak{F}}\rangle = {} & \sum_R j_{I_1} K_{I_1 I_2} (\partial_{I_2} f_R) \frac{\delta}{\delta f_R} |\hat{\mathfrak{F}}\rangle \\ & + \sum_{hI I_2 I_3 I_4} W^h_{I I_2 I_3 I_4} j_I \Bigg[\sum_R (\partial_{I_4} \partial_{I_3} \partial_{I_2} f_R) \frac{\delta}{\delta f_R} \\ & + \sum_{[\alpha\beta\gamma]} \sum_{RS} (\partial_{I_\alpha} \partial_{I_\beta} f_R)(\partial_{I_\gamma} f_S) \frac{\delta}{\delta f_S} \frac{\delta}{\delta f_R} \\ & + \sum_{RSR'} (\partial_{I_4} f_R)(\partial_{I_3} f_S)(\partial_{I_2} f_{R'}) \frac{\delta}{\delta f_{R'}} \frac{\delta}{\delta f_S} \frac{\delta}{\delta f_R} \\ & - \sum_{[\alpha\beta\gamma]} \sum_R F_{I_\alpha I'} j_{I'} (\partial_{I_\beta} \partial_{I_\gamma} f_R) \frac{\delta}{\delta f_R} \\ & - \sum_{[\alpha\beta\gamma]} \sum_{RS} F_{I_\alpha I'} j_{I'} (\partial_{I_\beta} f_R)(\partial_{I_\gamma} f_S) \frac{\delta}{\delta f_S} \frac{\delta}{\delta f_R} \\ & + \sum_{[\alpha\beta\gamma]} \sum_R F_{I_\alpha I'} F_{I_\beta I''} j_{I'} j_{I''} (\partial_{I_\gamma} f_R) \frac{\delta}{\delta f_R} \\ & + F_{I_4 I'} F_{I_3 I''} F_{I_2 I'''} j_{I'} j_{I''} j_{I'''} \Bigg] |\hat{\mathfrak{F}}\rangle, \end{aligned} \tag{3.3}$$

where the summation definition of $[\alpha, \beta, \gamma]$ is given in formula (2.7). A further inspection of the interaction terms in the bracket shows that the first term has to vanish for consistent three preon states, the second term leads to boson-fermion interactions, the third term is the fermionic self-coupling, the fourth term is the zero-order internal fermion interaction, the fifth term yields again boson-fermion interactions, while the sixth term is a first-order internal fermion interaction whereas the last term is a pure boson part. This specification demonstrates that a purely fermionic weak mapping procedure is not self-consistent but necessarily leads to the requirement to include boson states into the transformation from the beginning. Nevertheless, for the formation of gluons the inclusion of bosons is meaningless. Thus we omit these fermion-boson terms in (3.3) and take into account only the relevant fermion terms. In addition, for further simplification we consider only the lowest order fermion parts of (3.3) with respect to the polarization cloud. This yields with (2.1) and (2.5) for $\hat{\mathscr{H}}$ the approximate expression

$$\hat{\mathscr{H}} \Rightarrow \hat{\mathscr{H}}_{\mathrm{F}} := \sum_{RR'} K_{I_1 I_2}\, 3\, C_R^{I_2 A_1 A_2} R^{R'}_{I_1 A_1 A_2} f_{R'} \frac{\delta}{\delta f_R}$$
$$- \sum_{h[\alpha\beta\gamma]} \sum_{RR'} W^h_{I I_2 I_3 I_4} F_{I_\alpha I'}\, 6\, C_R^{I_\beta I_\gamma A} R^{R'}_{I I' A} f_{R'} \frac{\delta}{\delta f_R}$$
$$+ \sum_{hHRSR'} W^h_{I I_2 I_3 I_4}\, 3\, C_R^{I_2 A_1 A_2}\, 3\, C_S^{I_3 B_1 B_2}\, 3\, C_{R'}^{I_4 C_1 C_2}$$
$$\cdot R^H_{I A_1 A_2 B_1 B_2 C_1 C_2} f_H \frac{\delta}{\delta f_{R'}} \frac{\delta}{\delta f_S} \frac{\delta}{\delta f_R}, \tag{3.4}$$

where due to the appearance of R^H in the last term it is obvious that the effective four-fermion interaction of composite fermions is generated by the polarization cloud.

The first two terms in (3.4) describe the internal preon dynamics of a composite three-preon fermion state. Due to the antisymmetry of the dual functions $R^{R'}$ these terms can be rewritten into the more convenient form

$$\hat{\mathscr{H}}_{\mathrm{F}}^0 := \sum_{RR'} \sum_{\substack{W_1 W_2 W_3 \\ A_1 A_2 A_3}} \{[K_{W_1 A_1} \delta_{W_2 A_2} \delta_{W_3 A_3}$$
$$+ \delta_{W_1 A_1} K_{W_2 A_2} \delta_{W_3 A_3} + \delta_{W_1 A_1} \delta_{W_2 A_2} K_{W_3 A_3}]$$
$$+ \sum_{hI} [\delta_{A_1 W_1} \hat{W}^h_{W_2 A_2 A_3 I} F_{I W_3} + \delta_{A_2 W_2} \hat{W}^h_{W_1 A_1 A_3 I} F_{I W_3}$$
$$+ \delta_{A_3 W_3} \hat{W}^h_{W_1 A_1 A_2 I} F_{I W_2}]\}\, C_R^{A_1 A_2 A_3} R^{R'}_{W_1 W_2 W_3} f_{R'} \frac{\delta}{\delta f_R}, \tag{3.5}$$

where $\hat{W}$ is antisymmetrized with respect to the last three indices. Thus $\hat{\mathscr{H}}_{\mathrm{F}}$ can be written

$$\hat{\mathscr{H}}_{\mathrm{F}} = \hat{\mathscr{H}}_{\mathrm{F}}^0 + \hat{\mathscr{H}}_{\mathrm{F}}^{\mathrm{I}}, \tag{3.6}$$

where $\hat{\mathscr{H}}_{\mathrm{F}}^{\mathrm{I}}$ is given by the last term of (3.4), and in this approximation the fermion energy representation (3.3) yields

$$p_0 |\hat{\mathfrak{F}}\rangle = \hat{\mathscr{H}}_{\mathrm{F}} |\hat{\mathfrak{F}}\rangle, \tag{3.7}$$

i.e. we have obtained an energy representation for the effective composite fermion dynamics.

4. Evaluation of Composite Quark Dynamics

The evaluation of the effective composite fermion functional energy representation (3.7) can only be performed if the state functionals of the composite fermions are explicitly known. The undressed state functionals for leptons and quarks were given in [5]. In order to avoid a discussion of a possible SU(4) representation of composite leptons and quarks, we solely concentrate in this paper on effective composite quark dynamics, and postpone the discussion of lepton-lepton and lepton-quark effective interactions for future investigations.

According to [5] the quark functional state coefficients (wave functions!) read

$$C_{K,3}^{I_1 I_2 I_3} := U^{rr'r''} e^{i\boldsymbol{k}(\boldsymbol{r}+\boldsymbol{r}'+\boldsymbol{r}'')1/3}\, {}^{\varrho}\Theta^j_{\varkappa\varkappa'\varkappa''}$$
$$\cdot \left\{ \chi^l_{\alpha\alpha'\alpha''} \begin{pmatrix} 1\mathrm{s} \\ \boldsymbol{r}' - \boldsymbol{r}'' \end{pmatrix} \begin{pmatrix} 2\mathrm{p}_\varrho \\ \boldsymbol{r} - \frac{1}{2}(\boldsymbol{r}' + \boldsymbol{r}'') \end{pmatrix} \right\}_{as}, \tag{4.1}$$

where the notation is explained in detail in [5]. We only remark that the superspinor-isospinor part ${}^{\varrho}\Theta^j$ is symmetric, while the spinor-orbital part is antisymmetric. Furthermore, "color" is defined by the orbital angular momentum states $2\mathrm{p}_\varrho$. In a slight generalization of [5] which does not alter the results of the calculations in [5], we use the following expression for the spinor part χ^l of (4.1)

$$\chi^l_{\alpha\alpha'\alpha''} := \chi^l_\alpha Z_{\alpha'\alpha''} \tag{4.2}$$

with $Z := \gamma^5 C$, while χ^l_α will be determined later. With these states we evaluate (3.4), (3.5) and (3.6).

We begin with the analysis of $\hat{\mathscr{H}}_{\mathrm{F}}^0$. Due to (3.5) this operator decomposes into two parts

$$\hat{\mathscr{H}}_{\mathrm{F}}^0 = (\hat{\mathscr{H}}_{\mathrm{F}}^0)^{\mathrm{I}} + (\hat{\mathscr{H}}_{\mathrm{F}}^0)^{\mathrm{II}}, \tag{4.3}$$

where the first term denotes the kinetic energy while the second term denotes the internal interaction energy of the composite fermion state. Written with full indexing, these parts read

$$(\hat{\mathscr{H}}_{\mathrm{F}}^{0})^{\mathrm{I}} = \sum \int R\begin{pmatrix} \mathbf{x},\mathbf{y},\mathbf{z} & \varrho \\ r,u,v & \mathbf{k} \\ \alpha,\beta,\gamma & l \\ \varkappa,\nu,\mu & j\end{pmatrix}\left[K\begin{pmatrix}\mathbf{x},\mathbf{x}'\\ r,r'\\ \alpha,\alpha'\\ \varkappa,\varkappa'\end{pmatrix}\delta\begin{pmatrix}\mathbf{y},\mathbf{y}'\\ u,u'\\ \beta,\beta'\\ \nu,\nu'\end{pmatrix}\delta\begin{pmatrix}\mathbf{z},\mathbf{z}'\\ v,v'\\ \gamma,\gamma'\\ \mu,\mu'\end{pmatrix}\right.$$
$$+\,\delta\begin{pmatrix}\mathbf{x},\mathbf{x}'\\ r,r'\\ \alpha,\alpha'\\ \varkappa,\varkappa'\end{pmatrix}K\begin{pmatrix}\mathbf{y},\mathbf{y}'\\ u,u'\\ \beta,\beta'\\ \nu,\nu'\end{pmatrix}\delta\begin{pmatrix}\mathbf{z},\mathbf{z}'\\ v,v'\\ \gamma,\gamma'\\ \mu,\mu'\end{pmatrix}+\delta\begin{pmatrix}\mathbf{x},\mathbf{x}'\\ r,r'\\ \alpha,\alpha'\\ \varkappa,\varkappa'\end{pmatrix}\delta\begin{pmatrix}\mathbf{y},\mathbf{y}'\\ u,u'\\ \beta,\beta'\\ \nu,\nu'\end{pmatrix}K\left.\begin{pmatrix}\mathbf{z},\mathbf{z}'\\ v,v'\\ \gamma,\gamma'\\ \mu,\mu'\end{pmatrix}\right]$$
$$\cdot\, C\begin{pmatrix}\mathbf{x}',\mathbf{y}',\mathbf{z}' & \varrho' \\ r',u',v' & \mathbf{k}' \\ \alpha',\beta',\gamma' & l' \\ \varkappa',\nu',\mu' & j'\end{pmatrix} f\begin{pmatrix}\varrho\\ \mathbf{k}\\ l\\ j\end{pmatrix}\frac{\delta}{\delta f}\begin{pmatrix}\varrho'\\ \mathbf{k}'\\ l'\\ j'\end{pmatrix} \mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}z\,\mathrm{d}z'\,\mathrm{d}k\,\mathrm{d}k' \tag{4.4}$$

and

$$(\hat{\mathscr{H}}_{\mathrm{F}}^{0})^{\mathrm{II}} = \sum \int R\begin{pmatrix} \mathbf{x},\mathbf{y},\mathbf{z} & \varrho \\ r,u,v & \mathbf{k} \\ \alpha,\beta,\gamma & l \\ \varkappa,\nu,\mu & j\end{pmatrix}\left[\delta\begin{pmatrix}\mathbf{x}',\mathbf{x}\\ r',r\\ \alpha',\alpha\\ \varkappa',\varkappa\end{pmatrix}\hat{W}^{h}\begin{pmatrix}\mathbf{y},\mathbf{y}',\mathbf{z}',\mathbf{r}\\ u,u',v',s\\ \beta,\beta',\gamma',\delta\\ \nu,\nu',\mu',\varrho\end{pmatrix}F\begin{pmatrix}\mathbf{r},\mathbf{z}\\ s,v\\ \delta,\gamma\\ \varrho,\mu\end{pmatrix}\right.$$
$$+\,\delta\begin{pmatrix}\mathbf{y}',\mathbf{y}\\ u',u\\ \beta',\beta\\ \nu',\nu\end{pmatrix}\hat{W}^{h}\begin{pmatrix}\mathbf{x},\mathbf{x}',\mathbf{z}',\mathbf{r}\\ r,r',v',s\\ \alpha,\alpha',\gamma',\delta\\ \varkappa,\varkappa',\mu',\varrho\end{pmatrix}F\begin{pmatrix}\mathbf{r},\mathbf{z}\\ s,v\\ \delta,\gamma\\ \varrho,\mu\end{pmatrix}+\delta\begin{pmatrix}\mathbf{z}',\mathbf{z}\\ v',v\\ \gamma',\gamma\\ \mu',\mu\end{pmatrix}\hat{W}^{h}\begin{pmatrix}\mathbf{x},\mathbf{x}',\mathbf{y}',\mathbf{r}\\ r,r',u',s\\ \alpha,\alpha',\beta',\delta\\ \varkappa,\varkappa',\nu',\varrho\end{pmatrix}F\left.\begin{pmatrix}\mathbf{r},\mathbf{y}\\ s,u\\ \delta,\beta\\ \varrho,\nu\end{pmatrix}\right]$$
$$\cdot\, C\begin{pmatrix}\mathbf{x}',\mathbf{y}',\mathbf{z}' & \varrho' \\ r',u',v' & \mathbf{k}' \\ \alpha',\beta',\gamma' & l' \\ \varkappa',\nu',\mu' & j'\end{pmatrix} f\begin{pmatrix}\varrho\\ \mathbf{k}\\ l\\ j\end{pmatrix}\frac{\delta}{\delta f}\begin{pmatrix}\varrho'\\ \mathbf{k}'\\ l'\\ j'\end{pmatrix} \mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}z\,\mathrm{d}z'\,\mathrm{d}r\,\mathrm{d}k\,\mathrm{d}k'\,, \tag{4.5}$$

where the definitions of the various symbols are given in Section 1. As the states (4.1) are orthonormal the dual states R appearing in (4.4) and (4.5) are simply given by $R \equiv C^{\times}$. Thus all quantities appearing in (4.4) and (4.5) are well defined and (4.4), (4.5) can directly be evaluated. We perform this evaluation by using two approximations: According to [3, 4] we take into account only the leading term in the calculation of the sums over the auxiliary field parts $U^{rr'r''}$ and $S^{rr'r''}$, respectively, and the orbital exchange integrals. Furthermore, for the sum of the propagators of the subfields we use the approximate expression [3, 4]

$$\sum_{i=1}^{2} F^{i}(\mathbf{r},\mathbf{r}')_{\substack{\alpha\,\alpha'\\ \varkappa\,\varkappa'}} \equiv \eta^{-1}\, C^{-1}_{\alpha\alpha'}\,\gamma^{5}_{\varkappa\varkappa'}\,\delta(\mathbf{r}-\mathbf{r}')\,, \tag{4.6}$$

where $\varkappa$, $\varkappa'$ are combined superspinor-isospinor indices. Apart from these two approximations the evaluation of (4.4) and (4.5) can be done exactly. In particular all permutations, antisymmetrization etc. are fully taken into account. The corresponding calculations are straightforward, but due to the regard of all arising terms rather lengthy and tedious. Thus for the sake of brevity we cannot give any detail of these calculations here. We merely state the result. We obtain for $\hat{\mathscr{H}}_{\mathrm{F}}^{0}$ the following formula:

$$\hat{\mathscr{H}}_{\mathrm{F}}^{0} = -\sum_{jll'\varrho}\int {}^{\varrho}f_{l}^{j}(\mathbf{k})\,\chi_{\alpha}^{l\varkappa}\,\tfrac{1}{3}(\boldsymbol{\alpha}_{\alpha\alpha'}\cdot\mathbf{k}+m^{*}\beta_{\alpha\alpha'})\,\chi_{\alpha'}^{l'}$$
$$\cdot\,\frac{\delta}{\delta\,{}^{\varrho}f_{l'}^{j}(\mathbf{k}')}\,e^{-i(\mathbf{k}-\mathbf{k}')\mathbf{r}}\,\mathrm{d}r\,\mathrm{d}k\,\mathrm{d}k' \tag{4.7}$$

with

$$f\begin{pmatrix}\varrho\\ \boldsymbol{k}\\ j\\ l\end{pmatrix}\equiv {}^{\varrho}f_l^j(\boldsymbol{k})\,;\quad \frac{\delta}{\delta f}\begin{pmatrix}\varrho\\ \boldsymbol{k}\\ j\\ l\end{pmatrix}\equiv\frac{\delta}{\delta\,{}^{\varrho}f_l^j(\boldsymbol{k})}\,. \tag{4.8}$$

It is remarkable that the contribution of $(\hat{\mathscr{H}}_F^0)^{II}$ to (4.7) consists only in the addition of a mass term, so that the original preon mass term of (1.13), which appear in (4.4), are corrected to give the effective mass m^* of the composite fermions in (4.7). Since in this paper we are not interested in effective mass calculations of the composite fermions, we do not enter into a numerical discussion of m^*.

It is convenient to assume the spinor states χ_α^l to be eigenstates of the corresponding Dirac-operator in (4.7). Then we have $\chi_\alpha^l\equiv\chi_\alpha^l(\boldsymbol{k})$ and we can define the following operator expansion

$${}^{\varrho}f_\alpha^j(\boldsymbol{r}):=\sum_l\int\chi_\alpha^{l\times}(\boldsymbol{k})\,e^{-i\boldsymbol{k}\boldsymbol{r}}\,{}^{\varrho}f_l^j(\boldsymbol{k})\,\mathrm{d}k \tag{4.9}$$

and

$$\frac{\delta}{\delta\,{}^{\varrho}f_{\alpha'}^j(\boldsymbol{r})}:=\sum_{l'}\int\chi_{\alpha'}^{l'}(\boldsymbol{k}')\,e^{i\boldsymbol{k}'\boldsymbol{r}}\,\frac{\delta}{\delta\,{}^{\varrho}f_{l'}^j(\boldsymbol{k}')}\,\mathrm{d}k' \tag{4.10}$$

which yields for (4.7)

$$\hat{\mathscr{H}}_F^0=-\sum_{j,\varrho}\int{}^{\varrho}f_\alpha^j(\boldsymbol{r})\,\tfrac{1}{3}\,(\boldsymbol{\alpha}_{\alpha\alpha'}\cdot\boldsymbol{\nabla}+m^*\beta_{\alpha\alpha'})\,\frac{\delta}{\delta\,{}^{\varrho}f_{\alpha'}^j(\boldsymbol{r})}\,\mathrm{d}r\,, \tag{4.11}$$

i.e. this part of $\hat{\mathscr{H}}_F$ is the effective energy representation of free composite quarks, where ϱ is the color index, while j describes the superspinor-isospinor states.

We now discuss the contribution of effective quark-quark interactions to $\hat{\mathscr{H}}_F$. Written with full indexing this part reads

$$\begin{aligned}\hat{\mathscr{H}}_F^I=\sum_h\int W^h&\begin{pmatrix}\boldsymbol{r}, & \boldsymbol{x}, & \boldsymbol{y}, & \boldsymbol{z}\\ r, & r_2, & r_3, & r_4\\ \alpha, & \alpha_2, & \alpha_3, & \alpha_4\\ \varkappa, & \varkappa_2, & \varkappa_3, & \varkappa_4\end{pmatrix} C\left(\begin{array}{ccc|c}\boldsymbol{x}, & \boldsymbol{x}', & \boldsymbol{x}'' & a\\ r_2, & r_2', & r_2'' & \boldsymbol{q}\\ \alpha_2, & \alpha_2', & \alpha_2'' & l\\ \varkappa_2, & \varkappa_2', & \varkappa_2'' & j\end{array}\right) C\left(\begin{array}{ccc|c}\boldsymbol{y}, & \boldsymbol{y}', & \boldsymbol{y}'' & b\\ r_3, & r_3', & r_3'' & \boldsymbol{s}\\ \alpha_3, & \alpha_3', & \alpha_3'' & n\\ \varkappa_3, & \varkappa_3', & \varkappa_3'' & m\end{array}\right)\\ &\cdot C\left(\begin{array}{ccc|c}\boldsymbol{z}, & \boldsymbol{z}', & \boldsymbol{z}'' & c\\ r_4, & r_4', & r_4'' & \boldsymbol{q}'\\ \alpha_4, & \alpha_4', & \alpha_4'' & l'\\ \varkappa_4, & \varkappa_4', & \varkappa_4'' & j'\end{array}\right) R\left(\begin{array}{ccccccc|c}\boldsymbol{r}, & \boldsymbol{x}', & \boldsymbol{x}'', & \boldsymbol{y}', & \boldsymbol{y}'', & \boldsymbol{z}', & \boldsymbol{z}'' & d\\ r, & r_2', & r_2'', & r_3', & r_3'', & r_4', & r_4'' & \boldsymbol{p}\\ \alpha, & \alpha_2', & \alpha_2'', & \alpha_3', & \alpha_3'', & \alpha_4', & \alpha_4'' & n'\\ \varkappa, & \varkappa_2', & \varkappa_2'', & \varkappa_3', & \varkappa_3'', & \varkappa_4', & \varkappa_4'' & m'\end{array}\right)\\ &\cdot f\begin{pmatrix}d\\ \boldsymbol{p}\\ n'\\ m'\end{pmatrix}\frac{\delta}{\delta f}\begin{pmatrix}c\\ \boldsymbol{q}'\\ l'\\ j'\end{pmatrix}\frac{\delta}{\delta f}\begin{pmatrix}b\\ \boldsymbol{s}\\ n\\ m\end{pmatrix}\frac{\delta}{\delta f}\begin{pmatrix}a\\ \boldsymbol{q}\\ l\\ j\end{pmatrix}\mathrm{d}q\,\mathrm{d}q'\,\mathrm{d}p\,\mathrm{d}s\,\mathrm{d}r\,\mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}x''\,\mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}y''\,\mathrm{d}z\,\mathrm{d}z'\,\mathrm{d}z''\,.\end{aligned} \tag{4.12}$$

The "hard core" quark wave functions C appearing in this expression are given by (4.1). But apart from the complications which arise from the great number of variables and indices in (4.12), the exact evaluation of (4.12) depends upon the explicit knowledge of the polarization-cloud contribution R in (4.12). This contribution can be obtained from (2.11). Although the evaluation of (2.11) requires only ordinary integrations, it is a rather complicated problem due to the complicated Greenfunctions which are involved in these integrations. In order to get a first insight into the structure of effective quark dynamics and subsequent gluon formation, in the following we thus will make some plausible assumptions about these Greenfunctions which allow to circumvent these difficulties. For brevity we can only scetch the derivation of R from (2.11).

The resolution of the iteration scheme (2.11) can only be done in configuration space. After projection of (2.11) into configuration space we obtain the following formal expression for R

$$R^{\mathrm{H}}_{K_1\dots K_7}=\sum_{hh'}\sum_{\substack{K_1'\dots K_7'\\ K_1''\dots K_5''\,II'}}G_7\binom{K_1'\dots K_7'}{K_1\dots K_7}G_5\binom{K_1'',K_2'',K_3'',K_4'',K_5''}{K_1',K_2',K_3',K_4',I}\cdot R^{\mathrm{H}}_{K_1''K_2''I}\,W^h_{IK_5''K_4''K_3''}\,W^h_{I'K_7'K_6'K_5'}\,, \tag{4.13}$$

where due to the subsequent Hartree approximation of (4.12) explicit antisymmetrization is omitted. The functions G_7 and G_5 are the Greenfunctions of $(p_0-\mathscr{H}_0)$ in the corresponding five- or seven-preon sector, resp., and according to the preceding calculation we have $R^{\mathrm{H}}_{II'I''}\equiv C_{\mathrm{H}}^{*\,II'I''}$.

From (4.13) it follows that $R^{\mathrm{H},7}$ consists of a "hard core" which is given by the undressed quark wave functions (4.1) and a polarization cloud surrounding this hard core. In first approximation it is reasonable to assume that the polarization cloud function, or more precisely G_7 and G_5 are

i) factorized with respect to auxiliary field indices and the other remaining coordinates;
ii) scalar functions in the remaining coordinates;
iii) very concentrated functions in the relative space coordinates.

By means of these three properties it is possible to evaluate (4.13) without explicit knowledge of G_7 and G_5.

As a result of the application of i), ii) and iii) to (4.13) we obtain the following expression for R^{H}

$$R\left(\begin{array}{c|c} \boldsymbol{r}, \boldsymbol{x}', \boldsymbol{x}'', \boldsymbol{y}', \boldsymbol{y}'', \boldsymbol{z}', \boldsymbol{z}'' & d \\ r, r_2', r_2'', r_3', r_3'', r_4', r_4'' & \boldsymbol{p} \\ \alpha, \alpha_2', \alpha_2'', \alpha_3', \alpha_3'', \alpha_4', \alpha_4'' & m' \\ \varkappa, \varkappa_2', \varkappa_2'', \varkappa_3', \varkappa_3'', \varkappa_4', \varkappa_4'' & n' \end{array}\right) = S^{r r_2' r_2'' r_3' r_3'' r_4' r_4''} \quad (4.14)$$

$$\cdot \left({}^d\Theta^{m'}_{\varkappa\varkappa_2'\varkappa_2''}\, a_{\varkappa_3''\varkappa_3'}\, a_{\varkappa_4''\varkappa_4'}\right) \left(\chi^{n'}_{\alpha}\, Z_{\alpha_2'\alpha_2''}\, Z_{\alpha_3''\alpha_3'}\, Z_{\alpha_4''\alpha_4'}\right) \hat{\chi}_d$$

with

$$\hat{\chi}_d := \int \hat{G}_7(\boldsymbol{r}-\boldsymbol{r}_1', \boldsymbol{x}'-\boldsymbol{r}_2', \boldsymbol{x}''-\boldsymbol{r}_3, \boldsymbol{y}'-\boldsymbol{r}_4', \boldsymbol{y}''-\boldsymbol{r}_5', \boldsymbol{z}'-\boldsymbol{r}_6', \boldsymbol{z}''-\boldsymbol{r}_7') \cdot e^{i\boldsymbol{p}(\boldsymbol{r}_1'+\boldsymbol{r}_2'+\boldsymbol{r}_3')\,1/3}\, \chi(\boldsymbol{r}_2'-\boldsymbol{r}_3')\, \chi_d(\boldsymbol{r}_1'-\tfrac{1}{2}(\boldsymbol{r}_2'+\boldsymbol{r}_3'))$$
$$\cdot\, \delta(\boldsymbol{r}_3'-\boldsymbol{r}_4')\, \delta(\boldsymbol{r}_3'-\boldsymbol{r}_5')\, \delta(\boldsymbol{r}_3'-\boldsymbol{r}_6')\, \delta(\boldsymbol{r}_3'-\boldsymbol{r}_7')\, \mathrm{d}r_1'\, \mathrm{d}r_2'\, \mathrm{d}r_3'\, \mathrm{d}r_4'\, \mathrm{d}r_5'\, \mathrm{d}r_6'\, \mathrm{d}r_7', \quad (4.15)$$

where $\hat{G}_7$ results from a partial convolution of G_5 and G_7. The special form of $\hat{G}_7$ is not needed in the following provided we assume that ii) and iii) holds also for $\hat{G}_7$. The algebraic parts of (4.14) are exactly calculated due to ii), and the "hard core" wave functions are defined by

$$\chi(\boldsymbol{r}) := \begin{pmatrix} 1\mathrm{s} \\ \boldsymbol{r} \end{pmatrix}; \quad \chi_d(\boldsymbol{r}) := \begin{pmatrix} 2\mathrm{p}_d \\ \boldsymbol{r} \end{pmatrix} \quad (4.16)$$

according to (4.1). Furthermore, S is not explicitly needed, while ${}^d\Theta^{m'}$ and Z follow from (4.1) and a is defined by $a_{\varkappa\varkappa'} := \delta_{AA'}\,\delta_{\Lambda 1}\,\delta_{\Lambda 2}$ in superindices.

In preceding papers [4, 5] it was demonstrated that in the leading term approximation the contributions of exchange terms to the effective composite particle interactions vanish or can be neglected, respectively. We assume that this also applies to the problem under consideration, i.e. that we have to evaluate only the direct interactions. The omission of exchange terms is effected by using instead of antisymmetrized wave functions the pure product states in (4.1) and the non-antisymmetrized expression (4.14) for the dressed dual wave functions R^{H}. This kind of approximation is usually denoted by "Hartree"-approximation.

For the evaluation of (4.12) in the Hartree-approximation we have to observe that in contrast to (4.4) and (4.5) the algebraic part of W^h of (1.14) need not be partially antisymmetrized, i.e. instead of $\hat{W}^h$ in (4.4) and (4.5) we have to apply only W^h. According to (1.14) this algebraic part can be written

$$\sum_h V^h \binom{\alpha_1\alpha_2\alpha_3\alpha_4}{\varkappa_1\varkappa_2\varkappa_3\varkappa_4} = v_1(\varkappa_1\varkappa_2\varkappa_3\varkappa_4)\, v_2(\alpha_1\alpha_2\alpha_3\alpha_4) \quad (4.17)$$

with $\varkappa$ etc. as combined isospinor-superspinor index.

We now substitute the Hartree form of the wave functions (4.1) and (4.14), (4.15) into (4.12) and introduce the definitions

$$w_1(\underset{a}{j}, \underset{b}{m}, \underset{c}{j'}, \underset{d}{m'}) := v_1(\varkappa_1\varkappa_2\varkappa_3\varkappa_4)\, {}^a\Theta^{j}_{\varkappa_2\varkappa_2'\varkappa_2''}\, {}^b\Theta^{m}_{\varkappa_3\varkappa_3'\varkappa_3''}$$
$$\cdot\, {}^c\Theta^{j'}_{\varkappa_4\varkappa_4'\varkappa_4''}\, {}^d\Theta^{m'}_{\varkappa_1\varkappa_2'\varkappa_2''}\, a_{\varkappa_3''\varkappa_3'}\, a_{\varkappa_4''\varkappa_4'}, \quad (4.18)$$

$$w_2(l, n, l', n') := v_2(\alpha_1\alpha_2\alpha_3\alpha_4)\, \chi^{l}_{\alpha_2\alpha_2'\alpha_2''}\, \chi^{n}_{\alpha_3\alpha_3'\alpha_3''}\, \chi^{l'}_{\alpha_4\alpha_4'\alpha_4''}$$
$$\cdot\, \chi^{n'}_{\alpha_1}\, Z_{\alpha_2'\alpha_2''}\, Z_{\alpha_3''\alpha_3'}\, Z_{\alpha_4''\alpha_4'} \quad (4.19)$$

and

$$g(\eta) := \sum (-1)^r\, U^{r_2 r_2' r_2''}\, U^{r_3 r_3' r_3''}\, U^{r_4 r_4' r_4''}$$
$$\cdot\, S^{r r_2' r_2'' r_3' r_3'' r_4' r_4''} \quad (4.20)$$

After integration over $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{r}_4', \boldsymbol{r}_5', \boldsymbol{r}_6', \boldsymbol{r}_7'$ we obtain from (4.12) the expression

$$\hat{\mathscr{H}}^{\mathrm{I}}_{\mathrm{F}} = \sum_{\substack{jmj'm' \\ lnl'n' \\ abcd}} g(\eta)\, w_1(\underset{a}{j}, \underset{b}{m}, \underset{c}{j'}, \underset{d}{m'})\, w_2(l, n, l', n')$$
$$\cdot \int e^{i\boldsymbol{q}(\boldsymbol{r}+\boldsymbol{x}'+\boldsymbol{x}'')\,1/3}\, \chi(\boldsymbol{x}-\boldsymbol{x}')\, \chi_a(\boldsymbol{r}-\tfrac{1}{2}(\boldsymbol{x}'+\boldsymbol{x}''))\, e^{is(\boldsymbol{r}+\boldsymbol{y}'+\boldsymbol{y}'')\,1/3}\, \chi(\boldsymbol{y}'-\boldsymbol{y}'')$$
$$\cdot\, \chi_b(\boldsymbol{r}-\tfrac{1}{2}(\boldsymbol{y}'+\boldsymbol{y}''))\, e^{i\boldsymbol{q}'(\boldsymbol{r}+\boldsymbol{z}'+\boldsymbol{z}'')\,1/3}\, \chi(\boldsymbol{z}'-\boldsymbol{z}'')\, \chi_c(\boldsymbol{r}-\tfrac{1}{2}(\boldsymbol{z}'+\boldsymbol{z}''))\, e^{i\boldsymbol{p}(\boldsymbol{r}_1'+\boldsymbol{r}_2'+\boldsymbol{r}_3')\,1/3} \quad (4.21)$$
$$\cdot\, \hat{G}_7(\boldsymbol{r}-\boldsymbol{r}_1', \boldsymbol{x}'-\boldsymbol{r}_2', \boldsymbol{x}''-\boldsymbol{r}_3', \boldsymbol{y}'-\boldsymbol{r}_3', \boldsymbol{y}''-\boldsymbol{r}_3', \boldsymbol{z}'-\boldsymbol{r}_3', \boldsymbol{z}''-\boldsymbol{r}_3')\, \chi(\boldsymbol{r}_2'-\boldsymbol{r}_3')$$
$$\cdot\, \chi_d(\boldsymbol{r}_1'-\tfrac{1}{2}(\boldsymbol{r}_2'+\boldsymbol{r}_3'))\, {}^d f^{m'}_{n'}(\boldsymbol{p})\, {}^c\partial^{j'}_{l'}(\boldsymbol{q}')\, {}^b\partial^{m}_{n}(\boldsymbol{s})\, {}^a\partial^{j}_{l}(\boldsymbol{q})$$
$$\cdot\, \mathrm{d}p\, \mathrm{d}q\, \mathrm{d}q'\, \mathrm{d}s\, \mathrm{d}r_1'\, \mathrm{d}r_2'\, \mathrm{d}r_3'\, \mathrm{d}r\, \mathrm{d}x'\, \mathrm{d}x''\, \mathrm{d}y'\, \mathrm{d}y''\, \mathrm{d}z'\, \mathrm{d}z''$$

with

$$ {}^{a}\partial_{l}^{j}(\boldsymbol{r}) := \frac{\delta}{\delta\, {}^{a}f_{l}^{j}(\boldsymbol{r})}\,. \tag{4.22}$$

After various transformations, integrations etc. which for brevity we cannot explicitly discuss, in the leading term approximation the expression (4.21) goes over into

$$\hat{\mathscr{H}}_{\mathrm{F}}^{\mathrm{I}} = \sum_{\substack{jmj'm'\\ lnl'n'\\ abcd}} g(\eta)\, w_1(j,m,j',m')\, w_2(l,n,l',n')\, T_{abcd} \cdot \int e^{i(\boldsymbol{q}+\boldsymbol{s}+\boldsymbol{q}'-\boldsymbol{p})\boldsymbol{r}}\, {}^{d}f_{n'}^{m'}(\boldsymbol{p})\, {}^{c}\partial_{l'}^{j'}(\boldsymbol{q}')\, {}^{b}\partial_{n}^{m}(\boldsymbol{s})\, {}^{a}\partial_{l}^{j}(\boldsymbol{q}) \cdot \mathrm{d}p\, \mathrm{d}q\, \mathrm{d}q'\, \mathrm{d}s\, \mathrm{d}r\,. \tag{4.23}$$

The tensor T in (4.23) is defined by

$$T_{abcd} := \int \chi_a(\boldsymbol{s}')\, \chi_d(\boldsymbol{s}')\, \chi_b(\boldsymbol{u}'')\, \chi_c(\boldsymbol{v}'')\, e^{i\boldsymbol{s}\boldsymbol{u}''2/3}\, e^{i\boldsymbol{q}'\boldsymbol{v}''2/3} \cdot \hat{G}_7(\boldsymbol{u},\boldsymbol{u}',\boldsymbol{u}',\boldsymbol{u}'',\boldsymbol{u}'',\boldsymbol{v}'',\boldsymbol{v}'')\, \mathrm{d}u\, \mathrm{d}u'\, \mathrm{d}u''\, \mathrm{d}v''\, \mathrm{d}s'\,. \tag{4.24}$$

Observing that according to ii) $\hat{G}_7$ is a scalar we obtain in the leading term approximation

$$T_{abcd} = \delta_{ad}\, \delta_{bc}\, T\,, \tag{4.25}$$

where the numerical constant T can be included into $g(\eta)$. If furthermore (4.2) is substituted into (4.19) and (4.9), (4.10), (4.22) is taken into account the expression (4.23) yields

$$\hat{\mathscr{H}}_{\mathrm{F}}^{\mathrm{I}} = \sum_{\substack{jmj'm'\\ ab}} \hat{g}(\eta)\, w_1(\underset{a}{j},\underset{b}{m},\underset{b}{j'},\underset{a}{m'}) \cdot [\gamma^0_{\alpha\alpha_2} C_{\alpha_3\alpha_4} - (\gamma^0\gamma^5)_{\alpha\alpha_2} (\gamma^5 C)_{\alpha_3\alpha_4}] \cdot \int {}^{a}f_{\alpha}^{m'}(\boldsymbol{r})\, {}^{b}\partial_{\alpha_4}^{j'}(\boldsymbol{r})\, {}^{b}\partial_{\alpha_3}^{m}(\boldsymbol{r})\, {}^{a}\partial_{\alpha_2}^{j}(\boldsymbol{r})\, \mathrm{d}r\,. \tag{4.26}$$

The various isospinor-superspinor wave functions ${}^{a}\Theta^{j}$ etc. were defined and extensively discussed for quarks and leptons in [5]. Thus we can use the quark states for a straightforward calculation of (4.18), which yields

$$w_1(\underset{a}{j},\underset{b}{m},\underset{b}{j'},\underset{a}{m'}) = \delta_{jm'}\, a_{mj'}^{T}\,; \quad a = 1,2,3\,,\ b = 1\,,$$
$$w_1(\underset{a}{j},\underset{b}{m},\underset{b}{j'},\underset{a}{m'}) = \delta_{jm'}\, a_{mj'}\,; \quad a = 1,2,3\,,\ b = 2,3\,. \tag{4.27}$$

This result reflects the differences in the calculation due to different hypercolor of the Han-Nambu quarks. Yet in the final formula this hypercolor dependence drops out. This can be verified by substituting (4.27) in (4.26) and by a suitable rearrangement of indices, commutations etc. for the $b = 1$ expression. Then for (4.26) the following equivalent expression eventually results

$$\hat{\mathscr{H}}_{\mathrm{F}}^{\mathrm{I}} = \hat{g}(\eta) \sum_h V^h \binom{\alpha_1\alpha_2\alpha_3\alpha_4}{\varkappa_1\varkappa_2\varkappa_3\varkappa_4}\, \delta_{\varrho_1\varrho_2}\, \delta_{\varrho_3\varrho_4} \cdot \int {}^{\varrho_1}f_{\alpha_1}^{\varkappa_1}(\boldsymbol{r})\, {}^{\varrho_4}\partial_{\alpha_4}^{\varkappa_4}(\boldsymbol{r})\, {}^{\varrho_3}\partial_{\alpha_3}^{\varkappa_3}(\boldsymbol{r})\, {}^{\varrho_2}\partial_{\alpha_2}^{\varkappa_2}(\boldsymbol{r})\, \mathrm{d}r\,. \tag{4.28}$$

Together with (4.11) we thus obtain for (3.6)

$$\hat{\mathscr{H}}_{\mathrm{F}} = \int {}^{\varrho}f_{\alpha}^{\varkappa}(\boldsymbol{r})\, [\boldsymbol{\alpha}_{\alpha\alpha'} \cdot \boldsymbol{\nabla} + m^* \beta_{\alpha\alpha'}]\, \delta_{\varkappa\varkappa'}\, \delta_{\varrho\varrho'}\, {}^{\varrho'}\partial_{\alpha'}^{\varkappa'}(\boldsymbol{r})\, \mathrm{d}r + \hat{g}(\eta) \sum_h V^h \binom{\alpha_1\alpha_2\alpha_3\alpha_4}{\varkappa_1\varkappa_2\varkappa_3\varkappa_4}\, \delta_{\varrho_1\varrho_2}\, \delta_{\varrho_3\varrho_4} \cdot \int {}^{\varrho_1}f_{\alpha_1}^{\varkappa_1}(\boldsymbol{r})\, {}^{\varrho_4}\partial_{\alpha_4}^{\varkappa_4}(\boldsymbol{r})\, {}^{\varrho_3}\partial_{\alpha_3}^{\varkappa_3}(\boldsymbol{r})\, {}^{\varrho_2}\partial_{\alpha_2}^{\varkappa_2}(\boldsymbol{r})\, \mathrm{d}r\,. \tag{4.29}$$

If we further introduce "quark"-normal ordered functionals by

$$|\hat{\mathfrak{F}}\rangle = \exp \int {}^{\varrho}f_{\alpha}^{\varkappa}(\boldsymbol{r})\, F^{q}(\underset{\substack{\varkappa\\ \alpha}}{\boldsymbol{r}},\ \underset{\substack{\varkappa'\\ \alpha'}}{\boldsymbol{r}'})\, \delta_{\varrho\varrho'}\, {}^{\varrho'}f_{\alpha'}^{\varkappa'}(\boldsymbol{r})\, \mathrm{d}r\, \mathrm{d}r'\, |\mathfrak{G}\rangle \tag{4.30}$$

then we obtain from (4.29) the quark-normal transformed energy representation

$$p_0|\mathfrak{G}\rangle = \int {}^{\varrho}f_{\alpha}^{\varkappa}(\boldsymbol{r})\, [\boldsymbol{\alpha}_{\alpha\alpha'} \cdot \boldsymbol{\nabla} + m^* \beta_{\alpha\alpha'}] \cdot \delta_{\varkappa\varkappa'}\, \delta_{\varrho\varrho'}\, {}^{\varrho'}\partial_{\alpha'}^{\varkappa'}(\boldsymbol{r})\, \mathrm{d}r\, |\mathfrak{G}\rangle + \hat{g} \sum_h V^h \binom{\alpha_1\alpha_2\alpha_3\alpha_4}{\varkappa_1\varkappa_2\varkappa_3\varkappa_4}\, \delta_{\varrho_1\varrho_2}\, \delta_{\varrho_3\varrho_4} \cdot \int {}^{\varrho_1}f_{\alpha_1}^{\varkappa_1}(\boldsymbol{r})\, {}^{\varrho_4}d_{\alpha_4}^{\varkappa_4}(\boldsymbol{r})\, {}^{\varrho_3}d_{\alpha_3}^{\varkappa_3}(\boldsymbol{r})\, {}^{\varrho_2}d_{\alpha_2}^{\varkappa_2}(\boldsymbol{r})\, \mathrm{d}r\, |\mathfrak{G}\rangle \tag{4.31}$$

with an analogous definition of "quark" derivatives ${}^{\varrho}d_{\alpha}^{\varkappa}(\boldsymbol{r})$ to formula (1.12) for preon sources. By comparison with (1.11) we see that the effective quark dynamics is a nonlinear spinorfield dynamics with a global $SU(2) \times SU(3)$ invariance and first order derivatives. In contrast to (1.11) we need no higher order derivatives (and thus auxiliary fields) for regularization in the quark case, as regularization of (4.35) is achieved by the high energy formfactors which do not appear in the low energy limit treated here, but which have to be taken into account if divergencies occur in quark dynamical calculations which cover all energy ranges. We do not discuss this subtle problem but assume for the following that non-singular quark-antiquark bound states exist due to this inherent regularization.

5. Composite Gluons and Gluon Dynamics

The second step in our program is the formation of gluons out of quark-antiquark pairs which is achieved by means of the effective quark dynamics

(4.31). As, apart from regularization and an additional SU(3) global invariance, this quark dynamics is identical with the preon dynamics, we can proceed along the lines of preon-antipreon fusion if we study bound quark-antiquark pairs, provided we make allowance for the additional SU(3). The preon-antipreon fusion leading to a Yang-Mills dynamics was extensively studied in [4]. In the following we thus proceed in complete analogy to [4] and explicitly discuss only those passages where deviations from [4] appear.

In the formal notation of Sect. 1 the quark effective energy representation reads

$$p_0|\mathfrak{G}\rangle = \sum_{I_1 I_2} f_{I_1} K_{I_1 I_2} \partial_{I_2}|\mathfrak{G}\rangle + \sum_{I_1 I_2 I_3 I_4} f_{I_1} Q_{I_1 I_2 I_3 I_4} d_{I_4} d_{I_3} d_{I_2}|\mathfrak{G}\rangle =: \mathscr{H}_q|\mathfrak{G}\rangle \tag{5.1}$$

with

$$K_{I_1 I_2} := \delta_{F_1 F_2} \delta_{\Lambda_1 \Lambda_2} \delta_{\varrho_1 \varrho_2} [\boldsymbol{\alpha}_{\alpha_1 \alpha_2} \cdot \nabla + m^* \beta_{\alpha_1 \alpha_2}] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \tag{5.2}$$

and

$$Q_{I_1 I_2 I_3 I_4} := \delta_{F_1 F_2} \delta_{F_3 F_4} \delta_{\Lambda_1 \Lambda_2} \delta_{\Lambda_3 1} \delta_{\Lambda_4 2} \cdot [\gamma^0_{\alpha_1 \alpha_2} C_{\alpha_1 \alpha_2} - (\gamma^0 \gamma^5)_{\alpha_1 \alpha_2} (\gamma^5 C)_{\alpha_3 \alpha_4}] \cdot \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\, \delta(\boldsymbol{r}_1 - \boldsymbol{r}_3)\, \delta(\boldsymbol{r}_1 - \boldsymbol{r}_4) \tag{5.3}$$

which correspond to [4] (1.15), (1.16), (1.17). The indices have the following meaning:

α = spinor index ($\alpha = 1, 2, 3, 4$),
Λ = superspinor index ($\Lambda = 1, 2$),
F = flavor index (isospin) ($F = 1, 2$),
ϱ = color index ($\varrho = 1, 2, 3$).

We now transform to bound quark-antiquark pairs by

$$g_K = \sum_{I_1 I_2} \hat{C}_K^{I_1 I_2} f_{I_1} f_{I_2} \tag{5.4}$$

which yields

$$\mathscr{H}_q\left[f, \frac{\delta}{\delta f}\right] = \hat{\mathscr{H}}_q\left[g, \frac{\delta}{\delta g}\right]; \quad |\mathfrak{G}[f]\rangle = |\hat{\mathfrak{G}}[g]\rangle. \tag{5.5}$$

According to [4] (5.2), in the leading term approximation only the terms

$$\hat{\mathscr{H}}_q \approx (\mathscr{H}^0_{gg}) + (\mathscr{H}^3_{gg}) \tag{5.6}$$

survive while all other terms drop out. These terms explicitly read [4] (2.12), (2.13)

$$(\mathscr{H}^0_{gg}) = \sum_{\substack{I I' W W' \\ K K'}} \hat{R}^{K'}_{I I'} (K_{IW} \delta_{I'W'} + \delta_{IW} K_{I'W'}) \cdot \hat{C}_K^{WW'} g_{K'}\, \delta/\delta g_K - \tfrac{1}{2} \sum_{\substack{L_1 L_2 L_3 I I' \\ K K'}} \hat{R}^{K'}_{I I'} \{\hat{Q}_{I L_1 L_2 L_3} F^q_{L_3 I'}\}_{\mathrm{as}(I I')} \cdot \hat{C}_K^{L_1 L_2} g_{K'}\, \delta/\delta g_K \tag{5.7}$$

and

$$(\mathscr{H}^3_{gg}) = \sum_{\substack{L_1 L_2 L_3 I I' \\ K_1 K_2 K_3}} \hat{R}^{K_3}_{I I'} \{\hat{Q}_{I L_1 L_2 L_3} \hat{C}_{K_2}^{L_3 I'}\}_{\mathrm{as}(I I')} \cdot \hat{C}_{K_1}^{L_1 L_2} g_{K_3} \frac{\delta}{\delta g_{K_2}} \frac{\delta}{\delta g_{K_1}}. \tag{5.8}$$

In contrast to [4] we combine superspinor-color indices into one index $\lambda := (\varrho \Lambda)$ and not superspinor-flavor indices. This leads to

$$\delta_{\Lambda_3 1} \delta_{\Lambda_4 2} \delta_{\varrho_3 \varrho_4} = A_{\lambda_3 \lambda_4} \tag{5.9}$$

with

$$A_{\lambda \lambda'} := \begin{pmatrix} 0 & \mathbb{1} \\ 0 & 0 \end{pmatrix}, \tag{5.10}$$

where the submatrices of A have 3×3 dimension. Then $\hat{Q}$ reads [4] (2.10)

$$\begin{aligned} &\hat{Q}_{I L_1 L_2 L_3} \\ &= \delta_{F F_1} \delta_{F_2 F_3} \delta_{\lambda \lambda_1} \Gamma^5_{\lambda_2 \lambda_3} [\gamma^0_{\alpha \alpha_1} C_{\alpha_2 \alpha_3} - (\gamma^0 \gamma^5)_{\alpha \alpha_1} (\gamma^5 C)_{\alpha_2 \alpha_3}] \\ &\quad - \delta_{F F_2} \delta_{F_1 F_3} \delta_{\lambda \lambda_2} \Gamma^5_{\lambda_1 \lambda_3} [\gamma^0_{\alpha \alpha_2} C_{\alpha_1 \alpha_3} - (\gamma^0 \gamma^5)_{\alpha \alpha_2} (\gamma^5 C)_{\alpha_1 \alpha_3}] \\ &\quad + \delta_{F F_3} \delta_{F_1 F_2} \delta_{\lambda \lambda_3} \Gamma^5_{\lambda_2 \lambda_4} [\gamma^0_{\alpha \alpha_3} C_{\alpha_1 \alpha_2} - (\gamma^0 \gamma^5)_{\alpha \alpha_3} (\gamma^5 C)_{\alpha_1 \alpha_2}] \end{aligned} \tag{5.11}$$

with

$$\Gamma^5_{\lambda \lambda'} := \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \tag{5.12}$$

and the quark propagator is approximately given by [4] (3.24)

$$F^q(\boldsymbol{r}, \boldsymbol{r}')_{\substack{\alpha\ \alpha' \\ \varrho\ \varrho' \\ \Lambda\ \Lambda' \\ F\ F'}} \approx -\delta_{FF'} \Gamma^5_{\lambda \lambda'} C^{-1}_{\alpha \alpha'} \delta(\boldsymbol{r} - \boldsymbol{r}'). \tag{5.13}$$

For the gluon wave functions we make the ansatz

$$\hat{C}_K^{II'} := e^{i\boldsymbol{k}(\boldsymbol{r}+\boldsymbol{r}')1/2} \chi^{\sigma t}(\boldsymbol{r} - \boldsymbol{r}' | \boldsymbol{k})\, S^{\sigma}_{\alpha \alpha'} T^t_{\lambda \lambda'} \delta_{Fa} \delta_{F'a} \tag{5.14}$$

which has to be compared with [4] (3.2), (3.5b). The spin parts S^σ are symmetric and are given by

$$\{S^\sigma\}_s := \{\gamma_\mu C, \Sigma_{\mu\nu} C\} \tag{5.15}$$

the superspinor-color parts T^t are antisymmetric. The spin parts correspond to [4] (3.7), the superspinor-color parts T^t must be newly defined. Let $\{\hat\lambda_i;\ i = 1, \ldots, 8\}$ be the set of SU(3) generators in the Gell-Mann-Ne'eman representation, then we define the following set of T-matrices

$$T^1 := \begin{pmatrix} 0 & \hat\lambda_1 \\ -\hat\lambda_1 & 0 \end{pmatrix};\ T^2 := \begin{pmatrix} 0 & \hat\lambda_2 \\ \hat\lambda_2 & 0 \end{pmatrix};\ T^3 := \begin{pmatrix} 0 & \hat\lambda_3 \\ -\hat\lambda_3 & 0 \end{pmatrix};$$

$$T^4 := \begin{pmatrix} 0 & \hat\lambda_4 \\ -\hat\lambda_4 & 0 \end{pmatrix};\ T^5 := \begin{pmatrix} 0 & \hat\lambda_5 \\ \hat\lambda_5 & 0 \end{pmatrix};\ T^6 := \begin{pmatrix} 0 & \hat\lambda_6 \\ -\hat\lambda_6 & 0 \end{pmatrix};$$

$$T^7 := \begin{pmatrix} 0 & \hat\lambda_7 \\ \hat\lambda_7 & 0 \end{pmatrix};\ T^8 := \begin{pmatrix} 0 & \hat\lambda_8 \\ -\hat\lambda_8 & 0 \end{pmatrix}. \tag{5.16}$$

These matrices are antisymmetric and fulfill the commutation relations

$$T^t \Gamma^5 T^{t'} - T^{t'} \Gamma^5 T^t = 2i f_{tt't''} T^{t''}, \tag{5.17}$$

where f_{ikj} are the SU(3) structure constants, cf. Greiner [22]. The validity of these relations can easily be checked by means of the commutation relations for the generators $\hat\lambda_i$ themselves, and as we will see these relations are crucial for the formation of gluons.

If now the gluon wave functions (5.14) and the quark propagators (5.13) are substituted into (5.7) and (5.8), we can proceed analogously to [4], Sect. 3 and 4. For the detailed evaluation we resolve the general index K of the wave functions $\hat C_K$ in (4.14) into its various components and write for the source operators

$$^a g_t^\sigma(\mathbf{r}) \equiv g_K;\quad {}^a\mathscr{D}_t^\sigma(\mathbf{r}) \equiv \delta/\delta g_K, \tag{5.18}$$

where the indices have the following meaning:

a = flavor-state index ($a = 1, 2$),
t = color-state index ($t = 1, \ldots, 8$),
σ = spin-state index ($\sigma = \mu, \varrho\mu$),

and the spin state is referred to the symmetric representation (5.15). Furthermore, in the following the dual sets $\{\hat S^\sigma\}$ and $\{\hat T^t\}$ with respect to the sets $\{S^\sigma\}$ and $\{T^t\}$ are used.

The calculation of the first term of (5.7) is straightforward and we obtain

$$(\mathscr{H}_{gg}^0)^{\mathrm{I}} = \sum_{\sigma\sigma' t a} \int \hat S_{\alpha\alpha'}^{\sigma'}\, {}^a g_t^\sigma(\mathbf{r}) \cdot \left\{ \left[-\frac{i}{2} \alpha_{\alpha\varrho}^k \partial_k + m^* \beta_{\alpha\varrho} \right] \delta_{\alpha'\varrho'} + \delta_{\alpha\varrho} \left[-\frac{i}{2} \alpha_{\alpha'\varrho'}^k \partial_k + m^* \beta_{\alpha'\varrho'} \right] \right\} S_{\varrho\varrho'}^\sigma\, {}^a\mathscr{D}_t^\sigma(\mathbf{r})\, \mathrm{d}r, \tag{5.19}$$

where m^* is the effective quark mass which is degenerate with respect to color and flavor.

For the second term of (5.7) we obtain

$$(\mathscr{H}_{gg}^0)^{\mathrm{II}} = g^* \sum_{\substack{\sigma\sigma' \\ tt'a}} \int \hat S_{\alpha\alpha'}^{\sigma'} M_{\alpha\alpha'}^\sigma(t', t)\, {}^a g_{t'}^{\sigma'}(\mathbf{r})\, {}^a\mathscr{D}_t^\sigma(\mathbf{r})\, \mathrm{d}r \tag{5.20}$$

with an effective coupling constant g^* and with

$$M_{\alpha\alpha'}^\sigma(t', t) := \delta_{Fa}\, \delta_{F'a}\, \hat T_{\lambda\lambda'}^{t'} \{ \hat Q \begin{pmatrix} \alpha\alpha_1\alpha_2\alpha_3 \\ \lambda\lambda_1\lambda_2\lambda_3 \\ FF_1F_2F_3 \end{pmatrix} C_{\alpha_3\alpha'}^{-1}\, \Gamma_{\lambda_3\lambda'}^5\, \delta_{F_3F'} \}_{\mathrm{as}(\lambda\lambda')}\, T_{\lambda_1\lambda_2}^t\, S_{\alpha_1\alpha_2}^\sigma\, \delta_{F_1a}\, \delta_{F_2a}. \tag{5.21}$$

The evaluation of this expression yields

$$M_{\alpha\alpha'}^\sigma(t', t) = 2\delta_{t't} [-\gamma_{\alpha\alpha_1}^0 \delta_{\alpha'\alpha_2} - \delta_{\alpha\alpha_1} \gamma_{\alpha'\alpha_2}^0 + (\gamma^0\gamma^5)_{\alpha\alpha_1} \gamma_{\alpha'\alpha_2}^5 + \gamma_{\alpha\alpha_1}^5 (\gamma^0\gamma^5)_{\alpha'\alpha_2}]\, S_{\alpha_1\alpha_2}^\sigma. \tag{5.22}$$

Finally, we discuss the interaction term (5.8). For this term we obtain

$$(\mathscr{H}_{gg}^3) = \sum_{\substack{\sigma\sigma'\sigma'' \\ tt't''a}} G^* \int \hat S_{\alpha\alpha'}^{\sigma''}\, \hat T_{\varkappa\varkappa'}^{t''}\, {}^a g_{t''}^{\sigma''}(\mathbf{r})\, M_{\substack{\alpha\alpha' \\ \varkappa\varkappa'}}^{\sigma'\sigma}(t', t)\, {}^a\mathscr{D}_{t'}^{\sigma'}(\mathbf{r})\, {}^a\mathscr{D}_t^\sigma(\mathbf{r})\, \mathrm{d}r, \tag{5.23}$$

where G is a further effective coupling constant. G^* as well as g^* can be explicitly calculated, but in this investigation we are not interested in the special values of these constants. The kernel M is given by

$$M_{\substack{\alpha\alpha' \\ \varkappa\varkappa'}}^{\sigma'\sigma}(t', t) := \delta_{Fa}\, \delta_{F'a} \{ \hat Q \begin{pmatrix} \alpha\alpha_1\alpha_2\alpha_3 \\ \lambda\lambda_1\lambda_2\lambda_3 \\ FF_1F_2F_3 \end{pmatrix} S_{\alpha_3\alpha'}^{\sigma'}\, T_{\lambda_3\lambda'}^{t'}\, \delta_{F_3a}\, \delta_{F'a} \}_{\mathrm{as}(\lambda\lambda')}\, S_{\alpha_1\alpha_2}^\sigma\, T_{\lambda_1\lambda_2}^t\, \delta_{F_1a}\, \delta_{F_2a}. \tag{5.24}$$

The evaluation of this expression yields

$$M^{\sigma'\sigma}_{\substack{\alpha\alpha' \\ \varkappa\varkappa'}}(t', t) = \{(T^t \Gamma^5 T^{t'})_{\lambda\lambda'}\}_{\mathrm{as}(\lambda\lambda')} [(\gamma^0 S^{\sigma'} C S^{\sigma})_{\alpha\alpha'} - (\gamma^0 \gamma^5 S^{\sigma'} \gamma^5 C S^{\sigma})_{\alpha\alpha'}]_{\mathrm{sym}(\alpha\alpha')} . \tag{5.25}$$

With $\{(T^t \Gamma^5 T^{t'})_{\lambda\lambda'}\}_{\mathrm{as}(\lambda\lambda')} = (T^t \Gamma^5 T^{t'} - T^{t'} \Gamma^5 T^t)_{\lambda\lambda'}$ (5.26)

we can apply (5.17) and obtain

$$M^{\sigma'\sigma}_{\substack{\alpha\alpha' \\ \varkappa\varkappa'}}(t', t) = 2 i f_{tt't''} T^{t''}_{\varkappa\varkappa'} [(\gamma^0 S^{\sigma'} C S^{\sigma}) - (\gamma^0 \gamma^5 S^{\sigma'} \gamma^5 C S^{\sigma})]_{\alpha\alpha'} . \tag{5.27}$$

The results of these calculation can be summarized to give the following expression for (5.6)

$$\begin{aligned} \hat{\mathscr{H}}_q = \sum_{\sigma\sigma' t a} \int \hat{S}^{\sigma'}_{\alpha\alpha'} {}^a g^{\sigma'}_t(\boldsymbol{r}) \Big\{ \Big[-\frac{i}{2} \alpha^k_{\alpha\varrho} \partial_k + m^* \beta_{\alpha\varrho} \Big] \delta_{\alpha'\varrho'} + g^* [-\beta_{\alpha\varrho} \delta_{\alpha'\varrho'} + (\beta\gamma^5)_{\alpha\varrho} \gamma^5_{\alpha'\varrho'}] \\ + \delta_{\alpha\varrho} \Big[-\frac{i}{2} \alpha^k_{\alpha'\varrho'} \partial_k + m^* \beta_{\alpha'\varrho'} \Big] + g^* [-\delta_{\alpha\varrho} \beta_{\alpha'\varrho'} + \gamma^5_{\alpha\varrho} (\beta\gamma^5)_{\alpha'\varrho'}] \Big\} S^{\sigma}_{\varrho\varrho'} {}^a\mathscr{D}^{\sigma}_t(\boldsymbol{r}) \, \mathrm{d}r \\ + G^* \sum_{\substack{\sigma\sigma'\sigma'' \\ tt't''a}} 2 i f_{tt't''} \int {}^a g^{\sigma''}_{t''}(\boldsymbol{r}) \, {}^a\mathscr{D}^{\sigma'}_{t'}(\boldsymbol{r}) \, {}^a\mathscr{D}^{\sigma}_t(\boldsymbol{r}) \, \mathrm{d}r \cdot \hat{S}^{\sigma''}_{\alpha\alpha'} [(\gamma^0 S^{\sigma'} C S^{\sigma}) - (\gamma^0 \gamma^5 S^{\sigma'} \gamma^5 C S^{\sigma})]_{\alpha\alpha'} . \end{aligned} \tag{5.28}$$

Apart from the fact that the indices t refer to color and the flavor index is not involved at all in a dynamical coupling of the fields (dumb index), the operator (5.28) is identical with the Yang-Mills energy representation of [4] (3.31), (4.12), (5.2).

We now assume that by the choice of the original coupling constant and the masses in the preon field equation (1.1) the effective gluon mass, which results from the further evaluation of (5.28), can be forced to vanish. Then the evaluation of (5.28) is completely identical with that of (5.2) in [4], Sect. 5. In particular, we obtain the functional energy representation for the effective gluon dynamics as a non-abelian SU (3) field law. As in all calculations of [4], Sect. 5, only $\varepsilon_{tt't''}$ has to be replaced by $f_{tt't''}$ in order to obtain the corresponding results for (5.28), we do not repeat this procedures here.

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